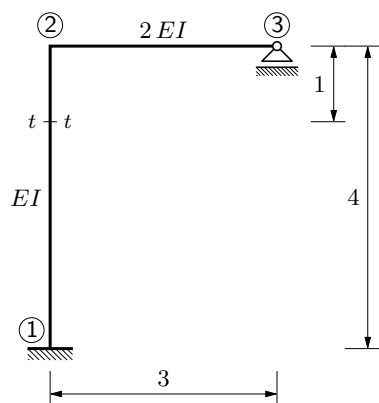
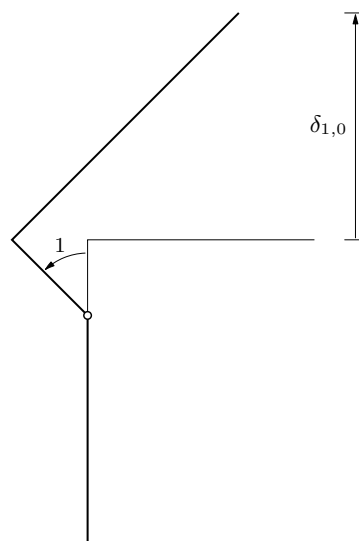
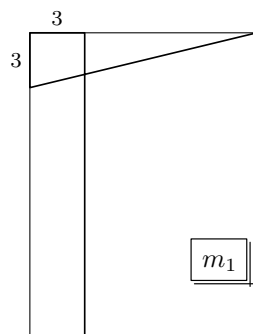
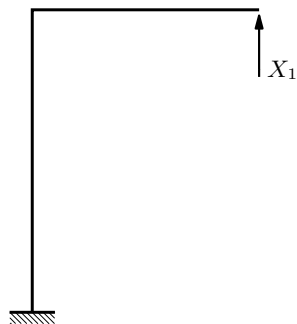




$$\eta_{M_{t-t}}$$

metoda sila:

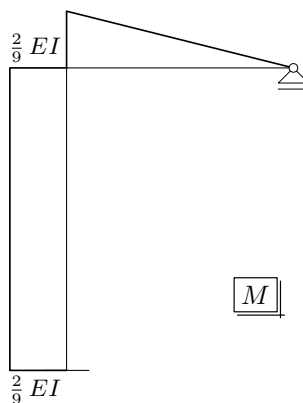


$$\delta_{1,1} X_1 + \delta_{1,0} = 0$$

$$\delta_{1,1} = \sum \int \frac{m_1^2}{EI} dx = \frac{1}{2EI} \left(\frac{1}{2} \cdot 3 \cdot 3 \right) \left(\frac{2}{3} \cdot 3 \right) + \frac{1}{EI} (3 \cdot 4) \cdot 3 = \frac{27}{2EI} \cdot 3$$

$$\frac{27}{2EI} \cdot 3 \cdot X_1 + 3 = 0 \quad \Rightarrow \quad \frac{27}{2EI} X_1 = -1 \quad \Rightarrow \quad X_1 = -\frac{2}{27} EI$$

$$M(x) = X_1 m_1(x)$$

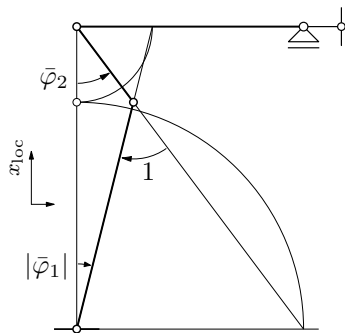


inženjerska metoda pomakā:

nepoznanice: $\varphi_2, u_{\{2,3\}}$

$$k_{\{1,2\}} = \frac{EI}{4}$$

$$k_{\{2,3\}} = \frac{2EI}{3}$$

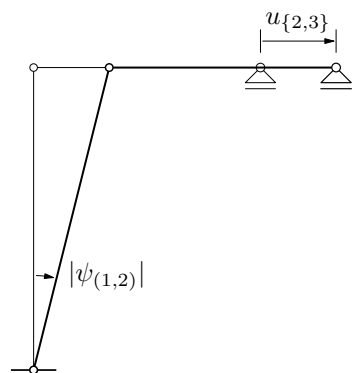


$$\bar{\varphi}_1 = -\frac{1}{4}, \quad \bar{\varphi}_2 = \frac{3}{4}$$

$$\begin{aligned} \bar{M}_{1,2} &= -4 k_{\{1,2\}} \bar{\varphi}_1 - 2 k_{\{1,2\}} \bar{\varphi}_2 \\ &= -4 \cdot \frac{EI}{4} \cdot \left(-\frac{1}{4}\right) - 2 \cdot \frac{EI}{4} \cdot \frac{3}{4} = -\frac{EI}{8} \end{aligned}$$

$$\begin{aligned} \bar{M}_{2,1} &= -4 k_{\{1,2\}} \bar{\varphi}_2 - 2 k_{\{1,2\}} \bar{\varphi}_1 \\ &= -4 \cdot \frac{EI}{4} \cdot \frac{3}{4} - 2 \cdot \frac{EI}{4} \cdot \left(-\frac{1}{4}\right) = -\frac{5EI}{8} \end{aligned}$$

$$\bar{M}_{2,3} = 0$$



$$\psi_{\{1,2\}} = -\frac{u_{\{2,3\}}}{4}, \quad \delta\psi_{\{1,2\}} = -\frac{\delta u_{\{2,3\}}}{4}$$

$$\begin{aligned} M_{1,2} &= 2 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} + \bar{M}_{1,2} \\ &= \frac{EI}{2} \varphi_2 + \frac{3EI}{8} u_{\{2,3\}} + \bar{M}_{1,2} \end{aligned}$$

$$\begin{aligned} M_{2,1} &= 4 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} + \bar{M}_{2,1} \\ &= EI \varphi_2 + \frac{3EI}{8} u_{\{2,3\}} + \bar{M}_{2,1} \end{aligned}$$

$$M_{2,3} = 3 k_{\{2,3\}} \varphi_2 = 2 EI \varphi_2$$

$$(1) \quad \sum_j M_{2,j} = 0; \quad M_{2,1} + M_{2,3} = 0$$

$$\left[EI \varphi_2 + \frac{3EI}{8} u_{\{2,3\}} - \frac{5EI}{8} \right] + 2 EI \varphi_2 = 0 \quad \implies \quad 3 \varphi_2 + \frac{3}{8} u_{\{2,3\}} = \frac{5}{8}$$

$$(2) \quad \sum W = 0; \quad (M_{1,2} + M_{2,1}) \delta\psi_{\{1,2\}} = 0$$

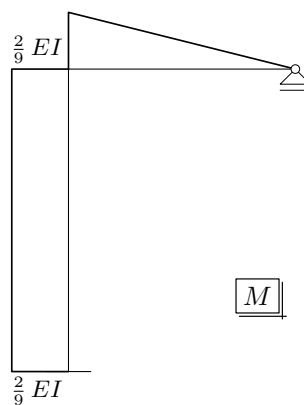
$$\left(\frac{3EI}{2} \varphi_2 + \frac{3EI}{4} u_{\{2,3\}} - \frac{3EI}{4} \right) \left(-\frac{\delta u_{\{2,3\}}}{4} \right) = 0 \quad \implies \quad \frac{3}{8} \varphi_2 + \frac{3}{16} u_{\{2,3\}} = \frac{3}{16}$$

$$(1) \text{ \& } (2) \quad \implies \quad \varphi_2 = 1/9, \quad u_{\{2,3\}} = 7/9$$

$$M_{1,2} = \frac{EI}{2} \cdot \frac{1}{9} + \frac{3EI}{8} \cdot \frac{7}{9} - \frac{EI}{8} = \frac{2}{9} EI$$

$$M_{2,1} = EI \cdot \frac{1}{9} + \frac{3EI}{8} \cdot \frac{7}{9} - \frac{5EI}{8} = -\frac{2}{9} EI$$

$$M_{2,3} = 2 EI \cdot \frac{1}{9} = \frac{2}{9} EI$$



iteracijski postupak:

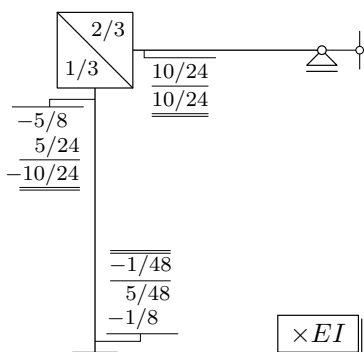
korak prvi: **Cross:**

$$k_2 = 4 k_{\{1,2\}} + 3 k_{\{2,3\}}$$

$$= 4 \frac{EI}{4} + 3 \frac{2EI}{3} = 3EI$$

$$\mu_{2,1} = \frac{4k_{\{1,2\}}}{k_2} = \frac{1}{3}$$

$$\mu_{2,3} = \frac{3k_{\{2,3\}}}{k_2} = \frac{2}{3}$$



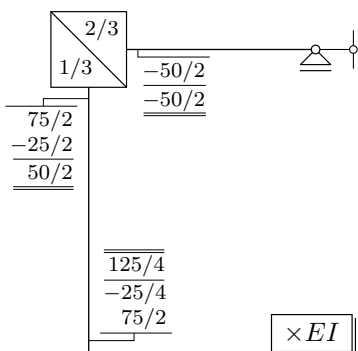
$$\begin{aligned} T_{2,1} &= \frac{1}{4} (M_{2,1} + M_{1,2}) \\ &= \frac{1}{4} \cdot \left(-\frac{10}{24} - \frac{1}{48} \right) \cdot EI \\ &= -\frac{1}{4} \cdot \frac{21}{48} \cdot EI \end{aligned}$$

$$R_1 = T_{2,1} = -\frac{1}{4} \cdot \frac{21}{48} \cdot EI$$

ili: korak drugi: **drugi Cross:**

$$\bar{u}_{2,3} = 100, \quad \bar{\psi}_{\{1,2\}} = -\frac{\bar{u}_{2,3}}{4} = -25$$

$$\overline{M}_{1,2} = \overline{M}_{2,1} = -6 k_{\{1,2\}} \overline{\psi}_{\{1,2\}} = -6 \cdot \frac{EI}{4} \cdot (-25) = \frac{75}{2} \cdot EI$$



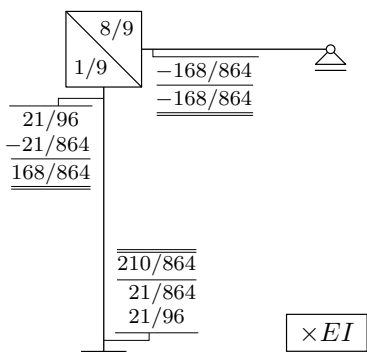
korak drugi: **Werner–Csonka:**

$$\overline{T}_{2,1} = -R_1 = \frac{1}{4} \cdot \frac{21}{48} \cdot EI$$

$$\overline{M}_{1,2} = \overline{M}_{2,1} = \frac{1}{2} \cdot \overline{T}_{2,1} \cdot 4 = \frac{21}{96} EI$$

$$k_2^W = k_{\{1,2\}} + 3k_{\{2,3\}} = \frac{9}{4}$$

$$\mu_{2,1} = \frac{k_{\{1,2\}}}{k_2^W} = \frac{1}{9}, \quad \mu_{2,3} = \frac{3k_{\{2,3\}}}{k_2^W} = \frac{3 \cdot 2/3}{3/4} = \frac{8}{9}$$



$$M_{2,3} = M_{2,3}^{\text{Cr}} + M_{2,3}^{\text{W}} = \left(\frac{10}{24} - \frac{168}{864} \right) EI = \frac{192}{864} EI = \frac{2}{9} EI$$

$$M_{2,1} = M_{2,1}^{\text{Cr}} + M_{2,1}^{\text{W}} = \left(-\frac{10}{24} + \frac{168}{864}\right) EI = -\frac{192}{864} EI = -\frac{2}{9} EI$$

$$M_{1,2} = M_{1,2}^{\text{Cr}} + M_{1,2}^{\text{W}} = \left(-\frac{1}{48} + \frac{210}{864}\right) EI = \frac{192}{864} EI = \frac{2}{9} EI$$

$$T'_{2,1} = \frac{1}{4} \cdot \left(\frac{50}{2} + \frac{125}{4} \right) \cdot EI = \frac{1}{4} \cdot \frac{225}{4} \cdot EI$$

$$R_2 = T'_{2,1} = \frac{1}{4} \cdot \frac{225}{4} \cdot EI$$

$$R_1 + \alpha R_2 = 0$$

$$-\frac{1}{4} \cdot \frac{21}{48} \cdot EI + \alpha \cdot \frac{1}{4} \cdot \frac{225}{4} \cdot EI = 0 \quad \Rightarrow \quad \alpha = \frac{7}{900}$$

$$M_{2,3} = M_{2,3}^{\text{Cr}} + \alpha M_{2,3}^{\text{Cr}^2} = \left(\frac{10}{24} - \frac{7}{900} \cdot \frac{50}{2} \right) EI = \frac{2}{9} EI$$

$$M_{2,1} = M_{2,1}^{\text{Cr}} + \alpha M_{2,1}^{\text{Cr}^2} = \left(-\frac{10}{24} + \frac{7}{900} \cdot \frac{50}{2}\right) EI = -\frac{2}{9} EI$$

$$M_{1,2} = M_{1,2}^{\text{Cr}} + \alpha M_{1,2}^{\text{Cr}^2} = \left(-\frac{1}{48} + \frac{7}{900} \cdot \frac{125}{4}\right) EI = \frac{2}{9} EI$$

crtanje utjecajne linije:

mjerilo duljina: 1 cm :: 1 m

$$\bar{\varphi}_1 = 3 \cdot \frac{2}{9} = 6/9$$

$$\tilde{\tilde{\varphi}}_1 = 3 \text{ cm}$$

$$\bar{\varphi}_2 = 1 \cdot \frac{2}{9} = 2/9$$

$$\tilde{\tilde{\varphi}}_2 = 1 \text{ cm}$$

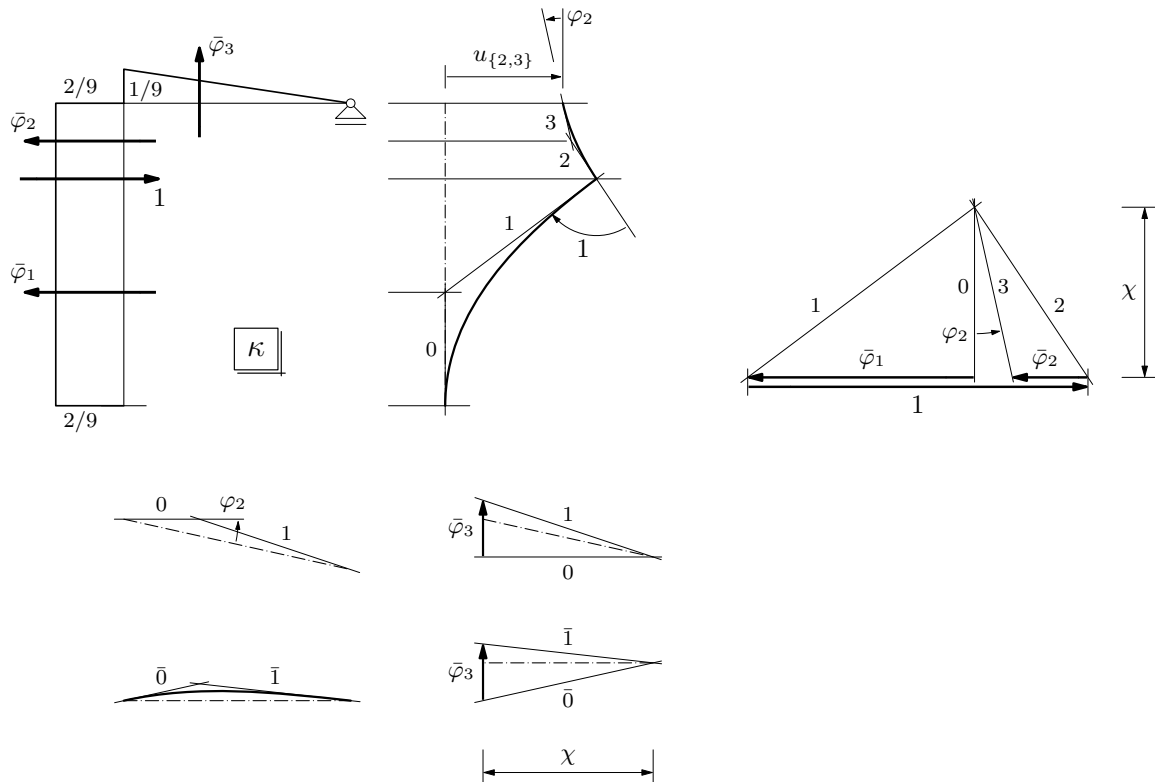
$$\bar{\varphi}_3 = \frac{1}{2} \cdot 3 \cdot \frac{1}{9} = 1/6$$

$$\tilde{\tilde{\varphi}}_3 = 0,75 \text{ cm}$$

mjerilo kutova: 1 cm :: 2/9

$$\tilde{\tilde{I}} = 4,5 \text{ cm}$$

$$\tilde{\tilde{\chi}} = 2,25 \text{ cm} \Rightarrow \chi = 1/2, n = 2$$



$$\eta = \frac{m}{n} \tilde{\tilde{\eta}} = \frac{1}{2} \tilde{\tilde{\eta}}$$

$$\tilde{\tilde{u}}_{\{2,3\}} = 15,5 \text{ mm} = 1,55 \text{ cm} \Rightarrow u_{\{2,3\}} = \frac{1}{2} \tilde{\tilde{u}}_{\{2,3\}} = 0,775 \approx 7/9$$

$$\varphi_2 = 1 - (\bar{\varphi}_1 + \bar{\varphi}_2) = 1 - (6/9 + 2/9) = 1/9$$