

GS 2. — popravni kolokvij (2023./2024.)

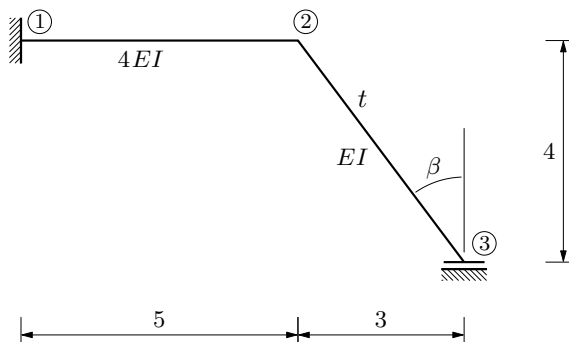
Zadatak 1.

Inženjerskom metodom pomakā nacrtajte dijagram momenata savijanja!

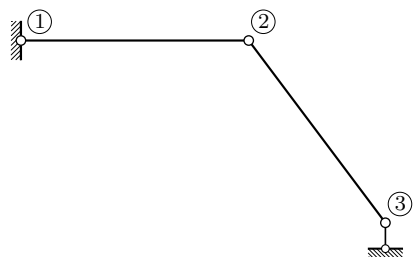
$$t = 20^{\circ} \text{ C (u štapu 2-3)}$$

$$EI = 20250 \text{ kNm}^2$$

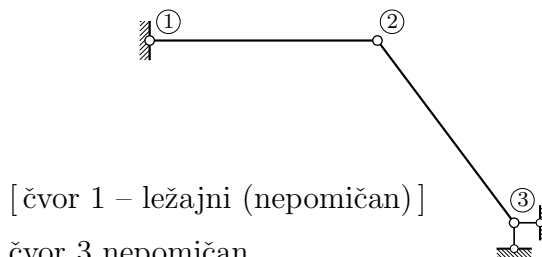
$$\alpha_t = 1 \cdot 10^{-5} \text{ K}^{-1}$$



zglobna shema i kinematička analiza zglobne sheme (dodavanje spoj(ev)a s podlogom):



$$s_{\min} = 2 \cdot 2 - 3 = 1$$

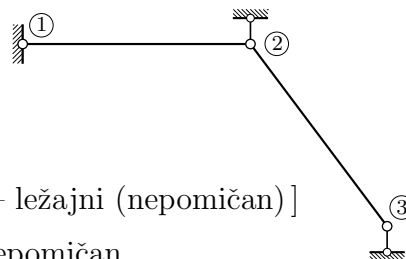


[čvor 1 – ležajni (nepomičan)]

čvor 3 nepomičan

→ čvor 2 nepomičan

ili



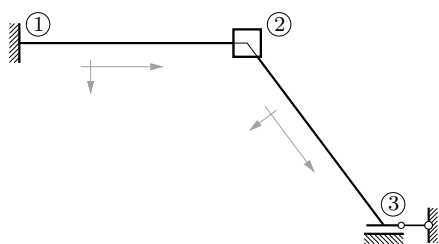
[čvor 1 – ležajni (nepomičan)]

čvor 2 nepomičan

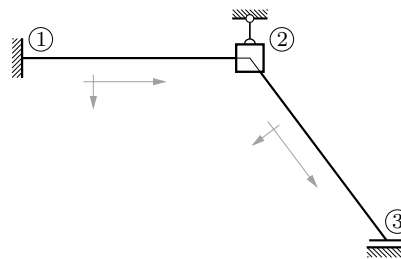
→ čvor 3 nepomičan

$$\Rightarrow s = s_{\min} = 1$$

osnovni sistem za inženjersku metodu pomakā:



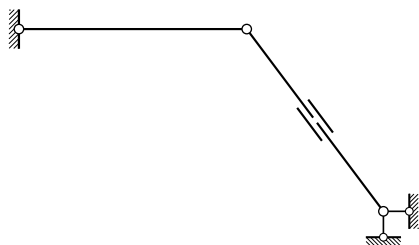
ili



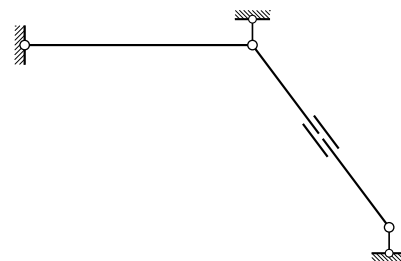
nepoznanice: φ_2 i u_3

nepoznanice: φ_2 i w_2

mehanizam za utjecaj promjene temperature:



ili

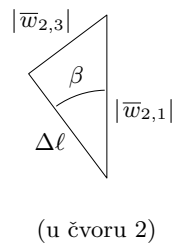
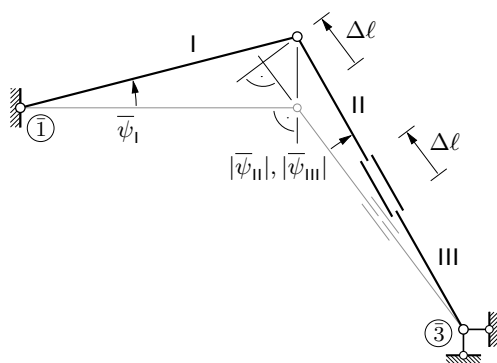


produljenje štapa 2-3 zbog jednolike promjene temperature:

$$\Delta \ell = \alpha_t t \ell_{\{2,3\}} = 1 \cdot 10^{-5} \cdot 20 \cdot 5 = 0,001$$

rješavanje pomoću lijevoga osnovnog sistema:

izračunavanje kutova $\bar{\psi}_{\{i,j\}}$ za utjecaj promjene temperature pomoću plana pomakā:



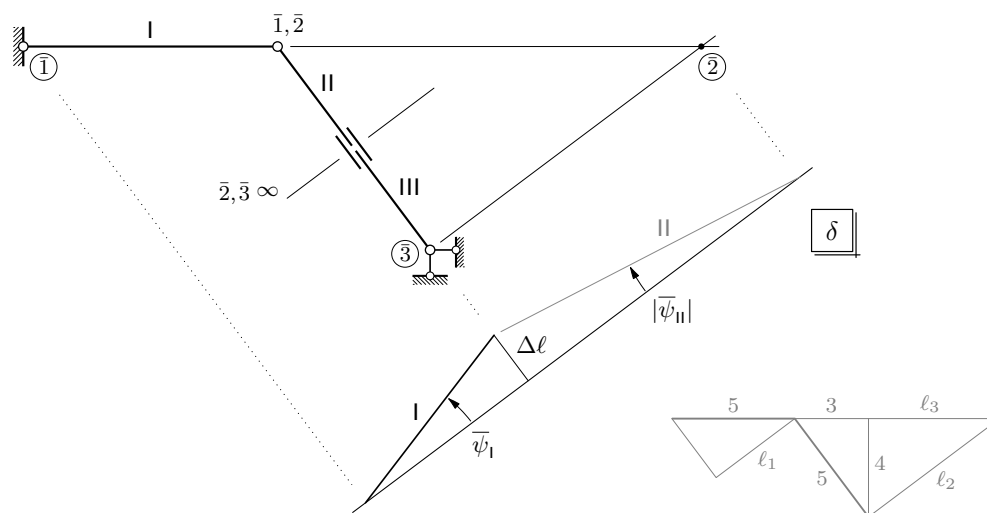
$$\frac{|w_{2,3}|}{\Delta \ell} = \frac{3}{4} \Rightarrow |w_{2,3}| = \frac{3}{4} \Delta \ell = 0,00075$$

$$\frac{\Delta \ell}{|\bar{w}_{2,1}|} = \frac{4}{5} \quad \Rightarrow \quad |\bar{w}_{2,1}| = \frac{5}{4} \Delta \ell = 0,00125$$

$$\bar{\psi}_I = \frac{|\bar{w}_{2,1}|}{\ell_{\{1,2\}}} = 0,00025 = \bar{\psi}_{\{1,2\}}$$

$$\bar{\psi}_{II} = \bar{\psi}_{III} = -\frac{|\bar{w}_{2,3}|}{\ell_{\{2,3\}}} = -0,00015 = \bar{\psi}_{\{2,3\}}$$

ili izračunavanje kutova $\bar{\psi}_{\{i,j\}}$ pomoću dijagrama projekcija pomakā na os štapa 2-3:



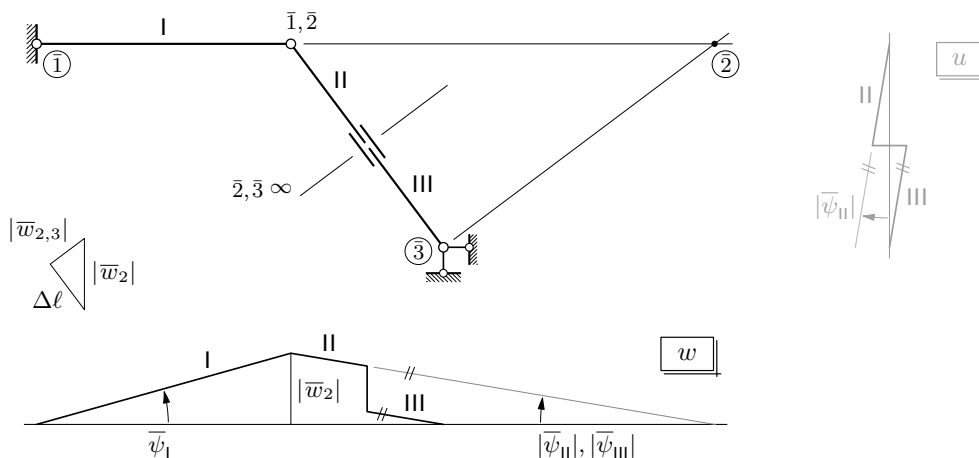
$$\bar{\psi}_I = \frac{\Delta \ell}{\ell_1} = 0,00025 = \bar{\psi}_{\{1,2\}}$$

$$\frac{\ell_1}{5} = \frac{4}{5} \quad \Rightarrow \quad \ell_1 = 4$$

$$\bar{\psi}_{II} = -\frac{\Delta \ell}{\ell_2} = -0,00015 = \bar{\psi}_{\{2,3\}}$$

$$\frac{\ell_2}{4} = \frac{5}{3} \quad \Rightarrow \quad \ell_2 = \frac{20}{3}$$

ili izračunavanje kutova $\bar{\psi}_{\{i,j\}}$ pomoću dijagrama projekcija pomakā na vertikalnu os:



(Treba ovdje naglasiti (prije je, pri crtanju plana pomakā bilo (ili barem trebalo biti) očito) da $\overrightarrow{\Delta\ell}$ (vektor duljine $\Delta\ell$ na osi štapa 2–3) nije pomak $\vec{\delta}_2$ čvora 2, tako da ni projekcije vektora $\overrightarrow{\Delta\ell}$ na vertikalnu i horizontalnu os nisu projekcije pomaka $\vec{\delta}_2$ na te osi. Pomak $\overrightarrow{\Delta\ell}$ samo je komponenta pomaka $\vec{\delta}_2$ na osi štapa 2–3, uzrokovana njegovim produljenjem. Štap 2–3 se zbog spoja sa štapom 1–2 i zaokreće (oko pola $\bar{2}$), pa $\vec{\delta}_2$ ima i komponentu $\vec{w}_{2,3}$ okomitu na os štapa 2–3. Kako čvor 2 „pripada” i štapu 1–2, pomiče se po okomici na os toga štapa, tako da zbroj komponentata $\overrightarrow{\Delta\ell}$ i $\vec{w}_{2,3}$ pomaka $\vec{\delta}_2$ mora biti na toj okomici (pogledajte i plan pomakā na najdonjoj slici na stranici 2), pa se $\vec{\delta}_2$ i njegova projekcija $\vec{w}_2$ na vertikalnu os (duljine $\bar{w}_2$) poklapaju, $\vec{\delta}_2 = \vec{w}_2$, a projekcija $\vec{u}_2$ na horizontalnu os iščezava, $\vec{u}_2 = \vec{0}$, $\bar{u}_2 = 0$.)

$$\frac{\Delta\ell}{|\bar{w}_2|} = \frac{4}{5} \quad \Rightarrow \quad |\bar{w}_2| = \frac{5}{4} \Delta\ell = 0,00125$$

$$\bar{\psi}_I = \frac{|\bar{w}_2|}{\ell_{\{1,2\}}} = 0,00025 = \bar{\psi}_{\{1,2\}}$$

$$\begin{aligned} \bar{\psi}_{II} = \bar{\psi}_{III} &= -\frac{|\bar{w}_2|}{3 + \ell_3} & \frac{\ell_3}{4} = \frac{4}{3} &\Rightarrow \ell_3 = \frac{16}{3} \\ &= -0,00015 = \bar{\psi}_{\{2,3\}} & & \text{(sivi dio donje slike na prethodnoj stranici)} \end{aligned}$$

(Dijagram projekcija pomakā na horizontalnu os nacrtan je potpunosti radi, iako je nepotreban i samo djelomice upotrebljiv — pomoću njega se $\bar{\psi}_I$ ne može odrediti.)

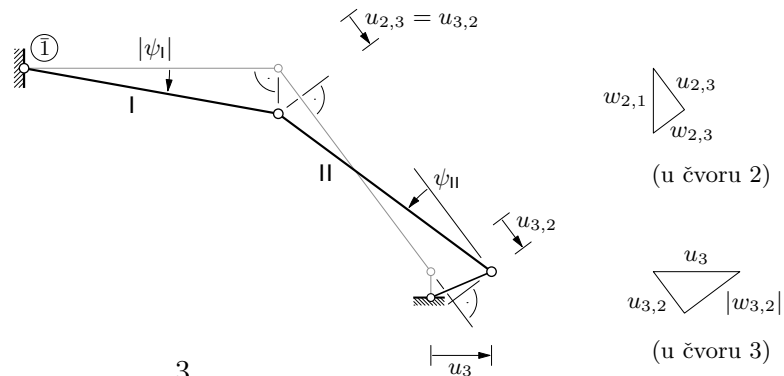
vrijednosti momenata upetosti:

$$\begin{aligned} \bar{M}_{1,2} &= \bar{M}_{2,1} = -6 k_{\{1,2\}} \bar{\psi}_{\{1,2\}} & k_{\{1,2\}} &= \frac{4EI}{\ell_{\{1,2\}}} = 16\,200 \text{ kNm} \\ &= -6 \cdot 16\,200 \cdot 0,00025 = -24,3 \text{ kNm} \\ \bar{M}_{2,3} &= \bar{M}_{3,2} = -6 k_{\{2,3\}} \bar{\psi}_{\{2,3\}} & k_{\{2,3\}} &= \frac{EI}{\ell_{\{2,3\}}} = 4\,050 \text{ kNm} \\ &= -6 \cdot 4\,050 \cdot (-0,00015) = 3,645 \text{ kNm} \end{aligned}$$

vrijednosti momenata na krajevima štapova:

$$\begin{aligned} M_{1,2} &= 2 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} + \bar{M}_{1,2} \\ M_{2,1} &= 4 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} + \bar{M}_{2,1} \\ M_{2,3} &= 4 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \bar{M}_{2,3} \\ M_{3,2} &= 2 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \bar{M}_{2,3} \end{aligned}$$

izračunavanje kutova $\psi_{i,j}$ za neovisni translacijski pomak $\vec{u}_3$ pomoću plana pomakā:
(zapetljano $\mathcal{E}$ ružno):



u čvoru 3:

$$\frac{u_{3,2}}{u_3} = \frac{3}{5} \Rightarrow u_{3,2} = \frac{3}{5} u_3$$

$$\frac{|w_{3,2}|}{u_3} = \frac{4}{5} \Rightarrow |w_{3,2}| = \frac{4}{5} u_3$$

u čvoru 2:

$$u_{2,3} = u_{3,2}$$

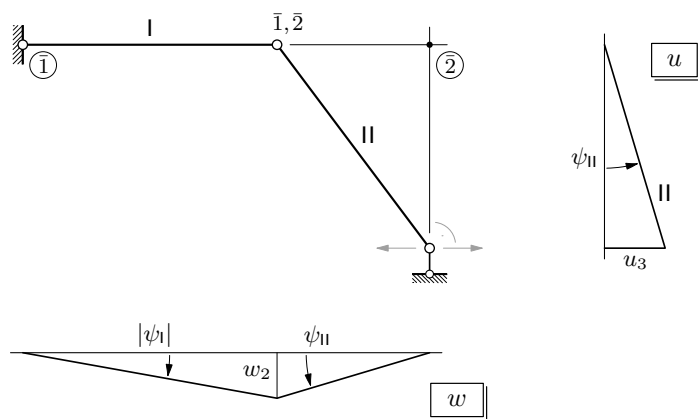
$$\frac{w_{2,3}}{u_{2,3}} = \frac{3}{4} \Rightarrow w_{2,3} = \frac{3}{4} u_{2,3} = \frac{9}{20} u_3$$

$$\frac{w_{2,1}}{u_{2,3}} = \frac{5}{4} \Rightarrow w_{2,1} = \frac{5}{4} u_{2,3} = \frac{3}{4} u_3$$

$$\psi_{\{1,2\}} = \psi_I = -\frac{w_{2,1}}{\ell_{\{1,2\}}} = -\frac{3}{20} u_3$$

$$\psi_{\{2,3\}} = \psi_{II} = \frac{w_{2,3} + |w_{3,2}|}{\ell_{\{2,3\}}} = \frac{1}{4} u_3$$

ili izračunavanje kutova $\psi_{i,j}$ pomoću dijagramā projekcijā pomakā na horizontalnu i na vertikalnu os:



$$\psi_{\{2,3\}} = \psi_{\parallel} = \frac{u_3}{4} = 0,25 u_3$$

$$w_2 = \psi_{\parallel} \cdot 3 = \frac{3}{4} u_3$$

$$\psi_{\{1,2\}} = \psi_{\perp} = -\frac{w_2}{5} = -\frac{3}{20} u_3 = -0,15 u_3$$

vrijednosti momenata na krajevima štapova (još jednom):

$$\begin{aligned} M_{1,2} &= 2 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} + \bar{M}_{1,2} \\ &= 2 \cdot 16\,200 \varphi_2 - 6 \cdot 16\,200 \cdot (-0,15 u_3) + (-24,3) \\ &= 32\,400 \varphi_2 + 14\,580 u_3 - 24,3 \end{aligned}$$

$$\begin{aligned} M_{2,1} &= 4 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} + \bar{M}_{2,1} \\ &= 4 \cdot 16\,200 \varphi_2 - 6 \cdot 16\,200 \cdot (-0,15 u_3) + (-24,3) \\ &= 64\,800 \varphi_2 + 14\,580 u_3 - 24,3 \end{aligned}$$

$$\begin{aligned} M_{2,3} &= 4 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \bar{M}_{2,3} \\ &= 4 \cdot 4\,050 \varphi_2 - 6 \cdot 4\,050 \cdot 0,25 u_3 + 3,645 \\ &= 16\,200 \varphi_2 - 6\,075 u_3 + 3,645 \end{aligned}$$

$$\begin{aligned} M_{3,2} &= 2 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \bar{M}_{2,3} \\ &= 2 \cdot 4\,050 \varphi_2 - 6 \cdot 4\,050 \cdot 0,25 u_3 + 3,645 \\ &= 8\,100 \varphi_2 - 6\,075 u_3 + 3,645 \end{aligned}$$

jednadžba ravnoteže momenata u čvoru 2:

$$\begin{aligned} -M_{2,1} + (-M_{2,3}) &= 0 \quad \Big| \quad \times (-1) \\ [64\,800 \varphi_2 + 14\,580 u_3 - 24,3] + [16\,200 \varphi_2 - 6\,075 u_3 + 3,645] &= 0 \\ 81\,000 \varphi_2 + 8\,505 u_3 - 20,655 &= 0 \end{aligned}$$

jednadžba rada na virtualnim pomacima:

(„vodeći” je virtualni pomak $\delta \vec{u}_3$ koji odgovara neovisnom translacijskom pomaku $\vec{u}_3$, pa su plan (virtualnih) pomaka i dijagrami projekcija (virtualnih) pomaka na koordinatne osi jednaki planu i dijagramima prikazanima na prethodnoj stranici, uz zamjene $u_3 \rightarrow \delta u_3$ i $\psi_i \rightarrow \delta \psi_i$, a veza je kutova $\delta \psi_{\{i,j\}}$ i orijentirane duljine δu_3 ista kao veza $\psi_{\{i,j\}}$ i u_3 :

$$\delta \psi_{\{1,2\}} = -\frac{3}{20} \delta u_3 = -0,15 \delta u_3 \quad \& \quad \delta \psi_{\{2,3\}} = \frac{\delta u_3}{4} = 0,25 \delta u_3$$

$$\begin{aligned}
& (M_{1,2} + M_{2,1}) \delta\psi_{\{1,2\}} + (M_{2,3} + M_{3,2}) \delta\psi_{\{2,3\}} = 0 \\
& (32\,400 \varphi_2 + 14\,580 u_3 - 24,3 + 64\,800 \varphi_2 + 14\,580 u_3 - 24,3) \cdot (-0,15 \delta u_3) \\
& \quad + (16\,200 \varphi_2 - 6\,075 u_3 + 3,645 + 8\,100 \varphi_2 - 6\,075 u_3 + 3,645) \cdot 0,25 \delta u_3 = 0 \\
& (-8\,505 \varphi_2 - 7\,411,5 u_3 + 9,102\,5) \delta u_3 = 0 \quad \forall \delta u_3 \\
& -8\,505 \varphi_2 - 7\,411,5 u_3 + 9,102\,5 = 0 \quad \Bigg| \quad \times (-1) \\
& 8\,505 \varphi_2 + 7\,411,5 u_3 - 9,102\,5 = 0
\end{aligned}$$

sustav jednačbi i njegovo rješenje:

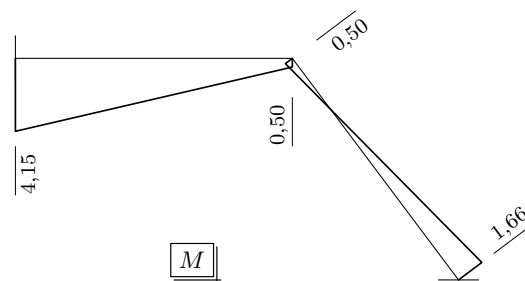
$$\begin{aligned}
81\,000 \varphi_2 + 8\,505 u_3 &= 20,655 \\
8\,505 \varphi_2 + 7\,411,5 u_3 &= 9,102\,5 \quad (\text{sustav je simetričan}) \\
\varphi_2 &= 0,000\,143\,3 \quad \mathcal{E} \quad u_3 = 0,001\,063\,7 \text{ m}
\end{aligned}$$

vrijednosti momenata na krajevima štapova (treći (i posljednji) put):

$$\begin{aligned}
M_{1,2} &= 32\,400 \cdot 0,000\,143\,3 + 14\,580 \cdot 0,001\,063\,7 - 24,3 = -4,148 \text{ kNm} \\
M_{2,1} &= 64\,800 \cdot 0,000\,143\,3 + 14\,580 \cdot 0,001\,063\,7 - 24,3 = 0,495 \text{ kNm} \\
M_{2,3} &= 16\,200 \cdot 0,000\,143\,3 - 6\,075 \cdot 0,001\,063\,7 + 3,645 = -0,496 \text{ kNm} \\
M_{3,2} &= 8\,100 \cdot 0,000\,143\,3 - 6\,075 \cdot 0,001\,063\,7 + 3,645 = -1,656 \text{ kNm}
\end{aligned}$$

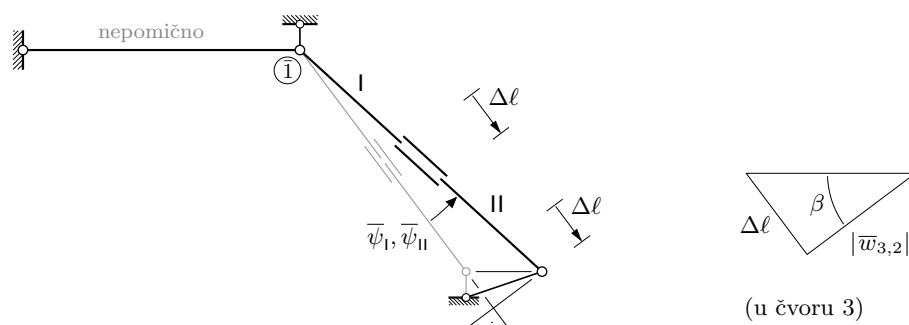
(„Neravnoteža” momenata u čvoru 2 ($M_{2,1} \neq |M_{2,3}|$) posljedica je pogrešaka zaokruživanja u međukoracima proračuna i njihova „gomilanja”).

dijagram momenata savijanja:



rješavanje pomoću desnoga osnovnog sistema:

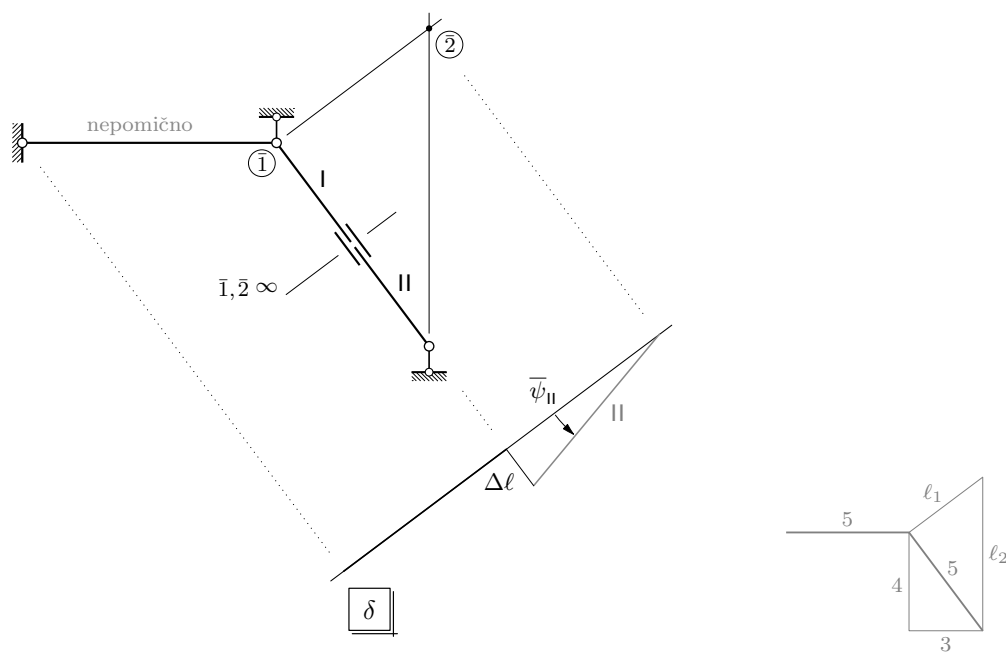
izračunavanje kuta $\bar{\psi}_{\{2,3\}}$ za utjecaj promjene temperature pomoću plana pomakā:



$$\frac{|\bar{w}_{3,2}|}{\Delta\ell} = \frac{4}{3} \quad \Rightarrow \quad |\bar{w}_{3,2}| = \frac{4}{3} \Delta\ell = 0,001\,333$$

$$\bar{\psi}_I = \bar{\psi}_{II} = \frac{|\bar{w}_{3,2}|}{\ell_{\{2,3\}}} = 0,000\,267 = \bar{\psi}_{\{2,3\}}$$

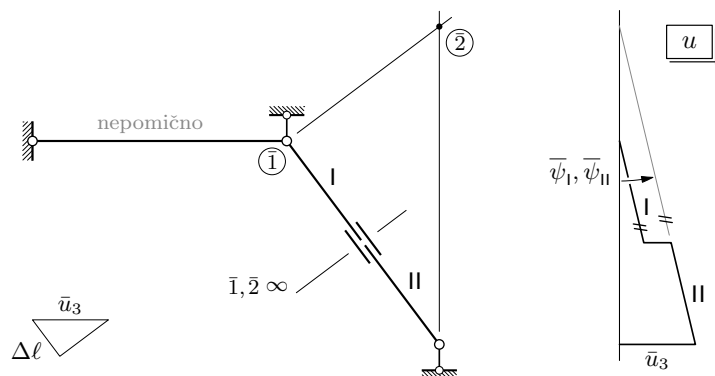
ili izračunavanje kuta $\bar{\psi}_{\{2,3\}}$ pomoću dijagrama projekcija pomakā na os štapa 2–3:



$$\bar{\psi}_{II} = \frac{\Delta\ell}{\ell_1} = 0,000\,267 = \bar{\psi}_{\{2,3\}}$$

$$\frac{\ell_1}{5} = \frac{3}{4} \quad \Rightarrow \quad \ell_1 = \frac{15}{4}$$

ili izračunavanje kuta $\bar{\psi}_{\{2,3\}}$ pomoću dijagrama projekcija pomaka na horizontalnu os:



$$\frac{\bar{u}_3}{\Delta\ell} = \frac{5}{3} \quad \Rightarrow \quad \bar{u}_3 = \frac{5}{3} \Delta\ell = 0,001\,667$$

$$\bar{\psi}_I = \bar{\psi}_{II} = \frac{\bar{u}_3}{\ell_2} = 0,000\,267 = \bar{\psi}_{\{2,3\}}$$

$$\frac{\ell_2}{5} = \frac{5}{4} \quad \Rightarrow \quad \ell_2 = \frac{25}{4}$$

(sivi dio donje slike na prethodnoj stranici)

vrijednosti momenata upetosti:

$$\bar{M}_{2,3} = \bar{M}_{3,2} = -6 k_{\{2,3\}} \bar{\psi}_{\{2,3\}} = -6 \cdot 4\,050 \cdot 0,000\,267 = -6,488\,1 \text{ kNm}$$

vrijednosti momenata na krajevima štapova:

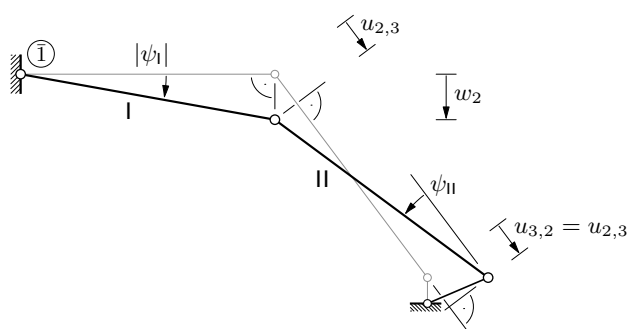
$$M_{1,2} = 2 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}}$$

$$M_{2,1} = 4 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}}$$

$$M_{2,3} = 4 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \bar{M}_{2,3}$$

$$M_{3,2} = 2 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \bar{M}_{2,3}$$

izračunavanje kutova $\psi_{i,j}$ za neovisni translacijski pomak $\vec{w}_2$ pomoću plana pomaka:
(zapetljano $\mathcal{E}$ ružno):



$$w_{2,1} = w_2 \begin{matrix} u_{2,3} \\ w_{2,3} \end{matrix} \quad (\text{u čvoru 2})$$

$$u_{3,2} \begin{matrix} w_{3,2} \end{matrix} \quad (\text{u čvoru 3})$$

(Plan je pomakā, dakako, jednak onom na gornjoj slici na stranici 5, ali s drugim „vodećim” pomakom.)

u čvoru 2:

$$w_{2,1} = w_2$$

$$\frac{u_{2,3}}{w_2} = \frac{4}{5} \quad \Rightarrow \quad u_{2,3} = \frac{4}{5} w_2$$

$$\frac{w_{2,3}}{w_2} = \frac{3}{5} \quad \Rightarrow \quad w_{2,3} = \frac{3}{5} w_2$$

u čvoru 3:

$$u_{3,2} = u_{2,3} = \frac{4}{5} w_2$$

$$\frac{|w_{3,2}|}{u_{3,2}} = \frac{4}{3} \quad \Rightarrow \quad |w_{3,2}| = \frac{4}{3} u_{3,2} = \frac{16}{15} w_2$$

$$\psi_{\{1,2\}} = \psi_{\perp} = -\frac{w_2}{\ell_{\{1,2\}}} = -\frac{w_2}{5}$$

$$\psi_{\{2,3\}} = \psi_{\parallel} = \frac{w_{2,3} + |w_{3,2}|}{\ell_{\{2,3\}}} = \frac{w_2}{3}$$

ili izračunavanje kutova $\psi_{i,j}$ pomoću dijagrama projekcijā pomakā na vertikalnu os:

dijagram je prikazan na donjoj slici na stranici 5

(na istoj slici prikazani dijagram projekcija pomakā na horizontalnu os sada nije potreban)

$$\psi_{\{1,2\}} = \psi_{\perp} = -\frac{w_2}{5} = -0,2 w_2$$

$$\psi_{\{2,3\}} = \psi_{\parallel} = \frac{w_2}{3} = 0,333\,3 w_2$$

vrijednosti momenata na krajevima štapova (još jednom):

$$\begin{aligned} M_{1,2} &= 2 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} \\ &= 2 \cdot 16\,200 \varphi_2 - 6 \cdot 16\,200 \cdot (-0,2 w_2) \\ &= 32\,400 \varphi_2 + 19\,440 w_2 \end{aligned}$$

$$\begin{aligned} M_{2,1} &= 4 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} \psi_{\{1,2\}} \\ &= 4 \cdot 16\,200 \varphi_2 - 6 \cdot 16\,200 \cdot (-0,2 w_2) \\ &= 64\,800 \varphi_2 + 19\,440 w_2 \end{aligned}$$

$$\begin{aligned}
M_{2,3} &= 4 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \overline{M}_{2,3} \\
&= 4 \cdot 4\,050 \varphi_2 - 6 \cdot 4\,050 \cdot 0,333\,3 w_2 - 6,488\,1 \\
&= 16\,200 \varphi_2 - 8\,100 w_2 - 6,488\,1
\end{aligned}$$

$$\begin{aligned}
M_{3,2} &= 2 k_{\{2,3\}} \varphi_2 - 6 k_{\{2,3\}} \psi_{\{2,3\}} + \overline{M}_{2,3} \\
&= 2 \cdot 4\,050 \varphi_2 - 6 \cdot 4\,050 \cdot 0,333\,3 w_2 - 6,488\,1 \\
&= 8\,100 \varphi_2 - 8\,100 w_2 - 6,488\,1
\end{aligned}$$

jednadžba ravnoteže momenata u čvoru 2:

$$\begin{aligned}
-M_{2,1} + (-M_{2,3}) &= 0 \quad \Big| \quad \times (-1) \\
[64\,800 \varphi_2 + 19\,440 w_2] + [16\,200 \varphi_2 - 8\,100 w_2 - 6,488\,1] &= 0 \\
81\,000 \varphi_2 + 11\,340 w_2 - 6,488\,1 &= 0
\end{aligned}$$

jednadžba rada na virtualnim pomacima:

(„vodeći” je virtualni pomak $\delta \vec{w}_2$ koji odgovara neovisnom translacijskom pomaku $\vec{w}_2$, pa su plan (virtualnih) pomaka i dijagrami projekcija (virtualnih) pomaka na koordinatne osi jednaki planu i dijagramima prikazanim na stranicama 9 i 5, uz zamjene $w_2 \rightarrow \delta w_2$ i $\psi_i \rightarrow \delta \psi_i$, a veza je kutova $\delta \psi_{\{i,j\}}$ i orijentirane duljine δw_2 ista kao veza $\psi_{\{i,j\}}$ i w_2 :

$$\begin{aligned}
\delta \psi_{\{1,2\}} &= -\frac{1}{5} \delta w_2 = -0,2 \delta w_2 \quad \& \quad \delta \psi_{\{2,3\}} = \frac{\delta w_2}{3} = 0,333\,3 \delta w_2 \\
(M_{1,2} + M_{2,1}) \delta \psi_{\{1,2\}} + (M_{2,3} + M_{3,2}) \delta \psi_{\{2,3\}} &= 0 \\
(32\,400 \varphi_2 + 19\,440 w_2 + 64\,800 \varphi_2 + 19\,440 w_2) \cdot (-0,2 \delta w_2) \\
+ (16\,200 \varphi_2 - 8\,100 w_2 - 6,488\,1 \\
+ 8\,100 \varphi_2 - 8\,100 w_2 - 6,488\,1) \cdot 0,333\,3 \delta w_2 &= 0 \\
(-11\,340 \varphi_2 - 13\,176 w_2 - 4,325\,4) \delta w_2 &= 0 \quad \forall \delta w_2 \\
-11\,340 \varphi_2 - 13\,176 w_2 - 4,325\,4 &= 0 \quad \Big| \quad \times (-1) \\
11\,340 \varphi_2 + 13\,176 w_2 + 4,325\,4 &= 0
\end{aligned}$$

sustav jednadžbi i njegovo rješenje:

$$\begin{aligned}
81\,000 \varphi_2 + 11\,340 w_2 - 6,488\,1 &= 0 \\
11\,340 \varphi_2 + 13\,176 w_2 + 4,325\,4 &= 0 \quad (\text{sustav je simetričan})
\end{aligned}$$

$$\varphi_2 = 0,000\,143\,3 \quad \mathcal{E}' \quad w_2 = -0,000\,451\,6 \text{ m}$$

vrijednosti momenata na krajevima štapova (treći (i posljednji) put):

$$M_{1,2} = 32\,400 \cdot 0,000\,143\,3 + 19\,440 \cdot (-0,000\,451\,6) = -4,136 \text{ kNm}$$

$$M_{2,1} = 64\,800 \cdot 0,000\,143\,3 + 19\,440 \cdot (-0,000\,451\,6) = 0,507 \text{ kNm}$$

$$M_{2,3} = 16\,200 \cdot 0,000\,143\,3 - 8\,100 \cdot (-0,000\,451\,6) - 6,488\,1 = -0,509 \text{ kNm}$$

$$M_{3,2} = 8\,100 \cdot 0,000\,143\,3 - 8\,100 \cdot (-0,000\,451\,6) - 6,488\,1 = -1,669 \text{ kNm}$$

(„Neravnoteža” momenata u čvoru 2 ($M_{2,1} \neq |M_{2,3}|$) i razlike u odnosu na vrijednosti momenata dobivene rješavanjem pomoću lijevoga osnovnog sistema posljedice su pogrešaka zaokruživanja u međukoracima proračuna i njihova „gomilanja”).

dijagram momenata savijanja — na stranici 7 (uz lagahna prilagođavanja vrijednosti)