

GS 2. — Prvi (i jedini) kolokvij (2024./2025.)

Zadatak 1. (Varijacija)

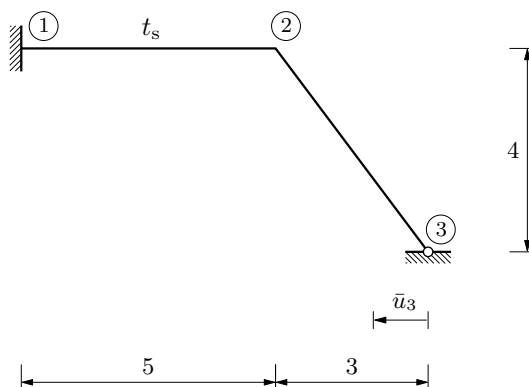
Inženjerskom metodom pomakā nacrtajte dijagram momenata savijanja!

$$t_s = -50^\circ \text{ C (na štapu 1-2)}$$

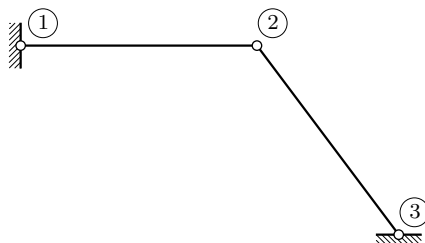
$$\bar{u}_3 = 2,5 \text{ mm}$$

$$EI = 20250 \text{ kNm}^2$$

$$\alpha_t = 1 \cdot 10^{-5} \text{ K}^{-1}$$



zglobna shema i nepoznanice (zapravo, nepoznanica):

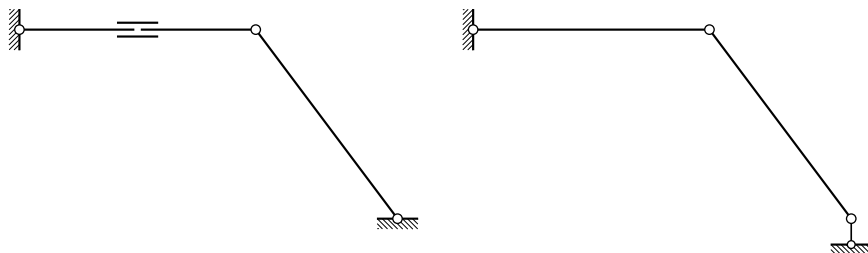


čvor 2 je s podlogom spojem s dva štapa osi kojih nisu na pravcu

$\Rightarrow$ čvor 2 je nepomičan

nepoznanica: (samo i jedino) φ_2

mehanizmi za utjecaj promjene temperature i za utjecaj prisilnoga pomaka:

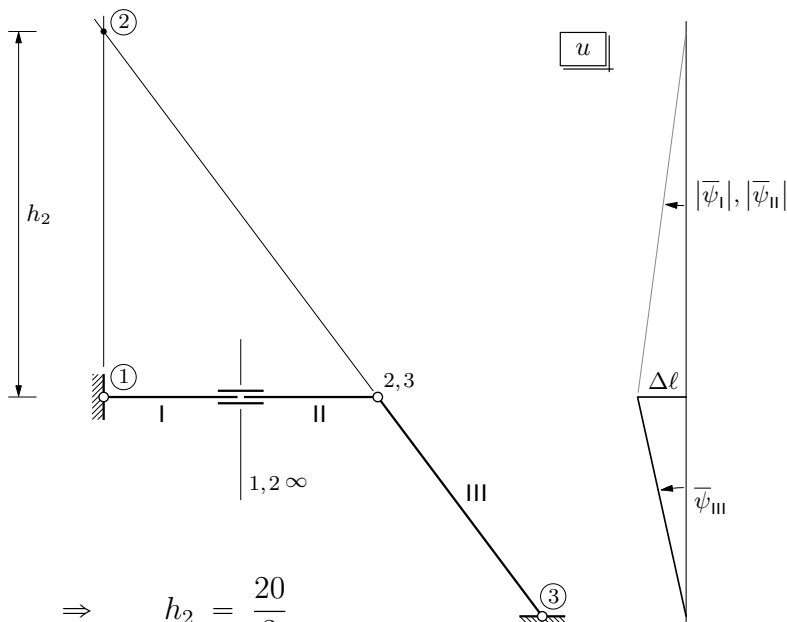


skraćenje štapa $\{1, 2\}$ zbog jednolike promjene temperature:

$$\Delta\ell = \alpha_t |t_s| \ell_{\{1,2\}} = 1 \cdot 10^{-5} \cdot 50 \cdot 5 = 0,0025$$

vrijednosti momenata upetosti za utjecaj promjene temperature:

izračunavanje kutova $\bar{\psi}_{\{i,j\}}$ pomoću dijagrama projekcija pomaka na horizontalnu os:

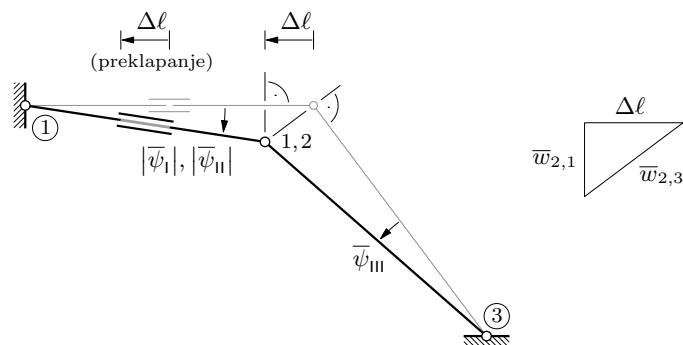


$$\frac{h_2}{5} = \frac{4}{3} \Rightarrow h_2 = \frac{20}{3}$$

$$\bar{\psi}_I = -\frac{\Delta\ell}{h_2} = -0,000375 = \bar{\psi}_{\{1,2\}}$$

$$\bar{\psi}_{III} = \frac{\Delta\ell}{4} = 0,000625 = \bar{\psi}_{\{2,3\}}$$

ili izračunavanje kutova $\bar{\psi}_{\{i,j\}}$ pomoću plana pomaka (zapetljanije, ali zornije):



$$\frac{\bar{w}_{2,1}}{\Delta\ell} = \frac{3}{4} \Rightarrow \bar{w}_{2,1} = \frac{3}{4} \Delta\ell = 0,001875$$

$$\frac{\bar{w}_{2,3}}{\Delta\ell} = \frac{5}{4} \Rightarrow \bar{w}_{2,3} = \frac{5}{4} \Delta\ell = 0,003125$$

$$\bar{\psi}_I = -\frac{\bar{w}_{2,1}}{\ell_{\{1,2\}}} = -0,000\,375 = \bar{\psi}_{\{1,2\}}$$

$$\bar{\psi}_{III} = \frac{\bar{w}_{2,3}}{\ell_{\{2,3\}}} = 0,000\,625 = \bar{\psi}_{\{2,3\}}$$

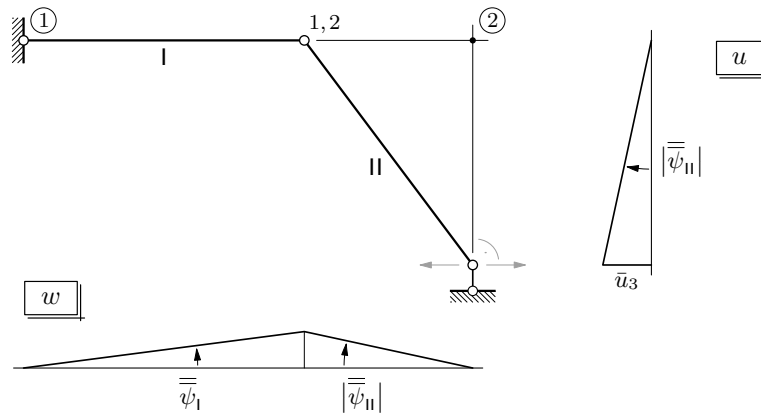
vrijednosti momenata upetosti:

$$\bar{M}_{1,2}(t_s) = \bar{M}_{2,1}(t_s) = -6\,k_{\{1,2\}}\,\bar{\psi}_{\{1,2\}} = -6 \cdot \frac{20\,250}{5} \cdot (-0,000\,375) = 9,112\,5\text{ kNm}$$

$$\bar{M}_{2,3}(t_s) = -3\,k_{\{2,3\}}\,\bar{\psi}_{\{2,3\}} = -3 \cdot \frac{20\,250}{5} \cdot 0,000\,625 = -7,593\,75\text{ kNm}$$

vrijednosti momenata upetosti za utjecaj prisilnoga pomaka:

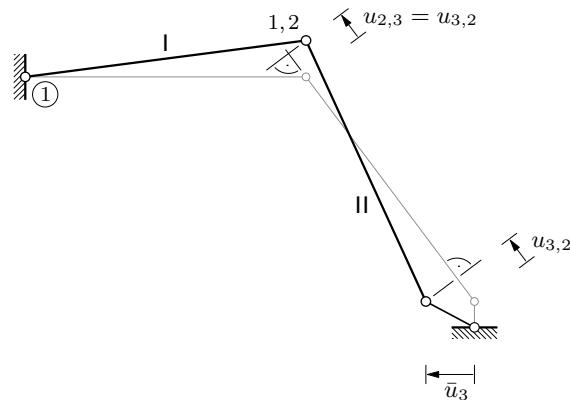
izračunavanje kutova $\bar{\bar{\psi}}_{\{i,j\}}$ pomoću dijagrama projekcija pomaka na horizontalnu i vertikalnu os:



$$\bar{\bar{\psi}}_{II} = -\frac{\bar{u}_3}{4} = -0,000\,625 = \bar{\bar{\psi}}_{\{2,3\}}$$

$$\bar{\bar{\psi}}_I \cdot 5 = |\bar{\bar{\psi}}_{II}| \cdot 3 \quad \Rightarrow \quad \bar{\bar{\psi}}_I = \frac{3}{5} |\bar{\bar{\psi}}_{II}| = 0,000\,375 = \bar{\bar{\psi}}_{\{1,2\}}$$

ili izračunavanje kutova $\bar{\bar{\psi}}_{\{i,j\}}$ pomoću plana pomaka (puno zapetljanije):



$\bar{\bar{\psi}}_I$ & $\bar{\bar{\psi}}_{II}$ — DZ! (za hrabr(ij)e)

vrijednosti momenata upetosti:

$$\bar{M}_{1,2}(\bar{u}_3) = \bar{M}_{2,1}(\bar{u}_3) = -6 k_{\{1,2\}} \bar{\bar{\psi}}_{\{1,2\}} = -6 \cdot \frac{20250}{5} \cdot 0,000375 = -9,1125 \text{ kNm}$$

$$\bar{M}_{2,3}(\bar{u}_3) = -3 k_{\{2,3\}} \bar{\bar{\psi}}_{\{2,3\}} = -3 \cdot \frac{20250}{5} \cdot (-0,000625) = 7,59375 \text{ kNm}$$

izrazi za vrijednosti ukupnih momenata na krajevima štapova:

$$M_{1,2} = 2 k_{\{1,2\}} \varphi_2 + \bar{M}_{1,2}(t_s) + \bar{M}_{1,2}(\bar{u}_3) = 2 k_{\{1,2\}} \varphi_2 + 9,1125 - 9,1125 = 2 k_{\{1,2\}} \varphi_2$$

$$M_{2,1} = 4 k_{\{1,2\}} \varphi_2 + \bar{M}_{2,1}(t_s) + \bar{M}_{2,1}(\bar{u}_3) = 4 k_{\{1,2\}} \varphi_2 + 9,1125 - 9,1125 = 4 k_{\{1,2\}} \varphi_2$$

$$M_{2,3} = 3 k_{\{2,3\}} \varphi_2 + \bar{M}_{2,3}(t_s) + \bar{M}_{2,3}(\bar{u}_3) = 3 k_{\{2,3\}} \varphi_2 - 7,59375 + 7,59375 = 3 k_{\{2,3\}} \varphi_2$$

jednadžba ravnoteže momenata u čvoru 2:

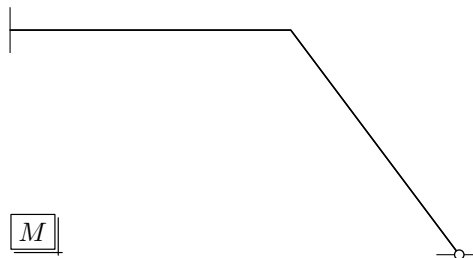
$$-M_{2,1} - M_{2,3} = 0 \quad \text{odnosno} \quad M_{2,1} + M_{2,3} = 0$$

$$4 k_{\{1,2\}} \varphi_2 + 3 k_{\{2,3\}} \varphi_2 = 0$$

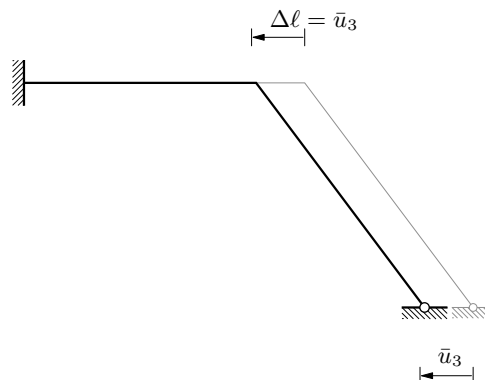
$$(4 k_{\{1,2\}} + 3 k_{\{2,3\}}) \varphi_2 = 0 \quad \Rightarrow \quad \varphi_2 = 0$$

$$\Rightarrow \quad M_{1,2} = M_{2,1} = M_{2,3} = 0$$

dijagram momenata savijanja:



skica „progibne linije” (zora radi):



[DZ — riješite zadatak ako je $t_s = +50^\circ \text{C}$!]