

GS 1. — 6. veljače 2024.

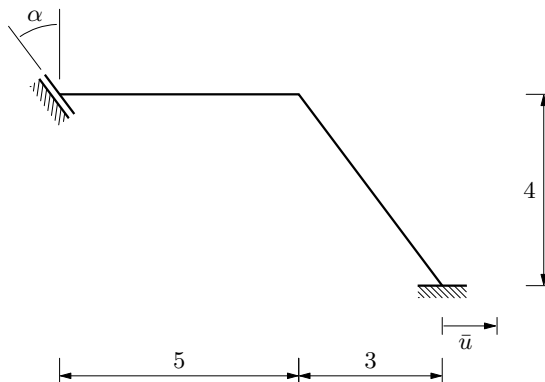
Zadatak 2.

Nacrtajte dijagrame unutarnjih sila! Provedite deformacijsku kontrolu!

$$\operatorname{tg} \alpha = 3/4$$

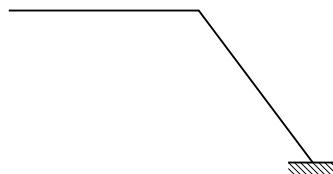
$$EI = 162000 \text{ kNm}^2$$

$$\bar{u} = 2 \text{ mm}$$



stupanj statičke neodređenosti i osnovni sistem(i):

neformalno, „pretvaranje” u (poznati) statički određeni sistem raskidanjem spojeva:
„slomljena” konzola:



uklonjen je ležaj koji oduzima dva stupnja slobode

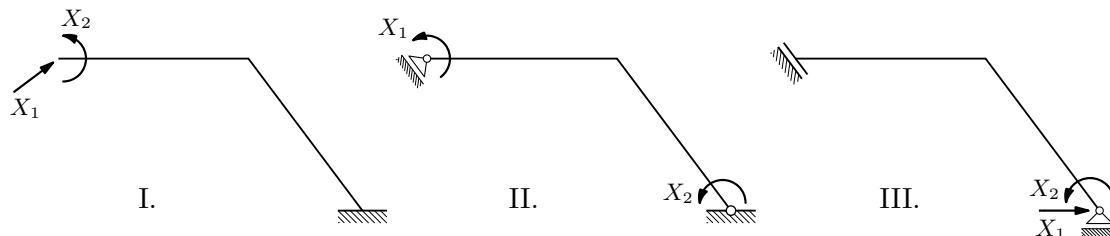
$\Rightarrow$ sistem je dva puta statički neodređen

formalnije, prebrojavanje i izračunavanje minimalnoga broja stupnjeva slobode:

$$S_{\min} = 3t - \ell = 3 \cdot 1 - (2 + 3) = -2$$

(te, potom, oblikovanje suvisloga statički određenog osnovnog sistema)

osnovni sistemi:



sustav jednačbi kompatibilnosti i vrijednosti momenata savijanja:

opći oblik sustava jednačbi kompatibilnosti:

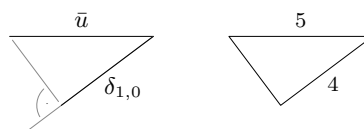
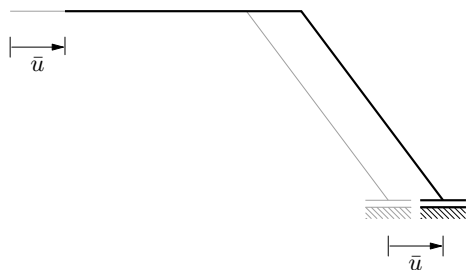
$$\delta_{1,1} X_1 + \delta_{1,2} X_2 + \delta_{1,0} = \bar{\delta}_1$$

$$\delta_{2,1} X_1 + \delta_{2,2} X_2 + \delta_{2,0} = \bar{\delta}_2$$

osnovni sistem I.:

$$\bar{\delta}_1 = 0 \quad \mathcal{E}' \quad \bar{\delta}_2 = 0 \quad (\text{nije raskinut spoj na mjestu i na pravcu prisilnoga pomaka})$$

$\delta_{i,0}$ pomoću plana pomakā:

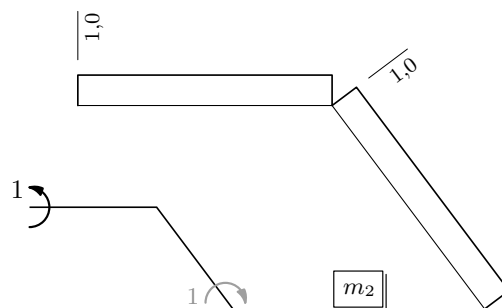
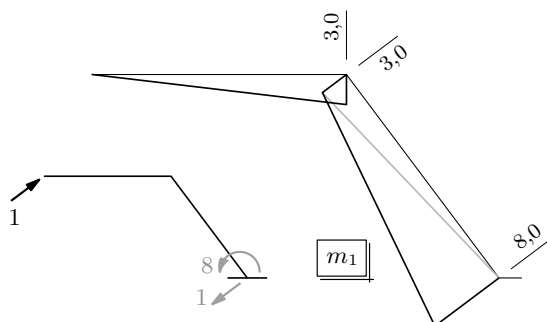


$$\frac{\delta_{1,0}}{\bar{u}} = \frac{4}{5}$$

$$\delta_{1,0} = \frac{4}{5} \bar{u} = \frac{4}{5} \cdot 0,002 = 0,0016$$

$$\delta_{2,0} = 0$$

dijagrami m_i i koeficijenti popustljivosti $\delta_{i,j}$:



$$\delta_{i,j} = \sum \int \frac{m_j(s) m_i(s)}{EI(s)} ds$$

$$\delta_{1,1} = \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 3,0 \cdot 5 \right) \left(\frac{2}{3} \cdot 3,0 \right) + \left(\frac{1}{2} \cdot 3,0 \cdot 5 \right) \left(\frac{2}{3} \cdot 3,0 + \frac{1}{3} \cdot 8,0 \right) + \left(\frac{1}{2} \cdot 8,0 \cdot 5 \right) \left(\frac{1}{3} \cdot 3,0 + \frac{2}{3} \cdot 8 \right) \right] = 1,09053 \cdot 10^{-3}$$

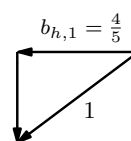
$$\delta_{2,2} = \frac{2}{EI} \left[(1,0 \cdot 5) \cdot 1,0 \right] = 6,17284 \cdot 10^{-5}$$

$$\delta_{1,2} = \delta_{2,1} = \frac{1}{EI} \left[(1,0 \cdot 5) \left(\frac{1}{2} \cdot 3,0 \right) (-1) + (1,0 \cdot 5) \left(\frac{1}{2} \cdot (3,0 + 8,0) \right) (-1) \right] = -2,16049 \cdot 10^{-4}$$

$\delta_{i,0}$ pomoću virtualnoga rada:

$$\delta_{1,0} = -b_{h,1} \bar{u} = -\left(-\frac{4}{5} \cdot 0,002 \right) = 0,0016$$

$$\delta_{2,0} = -b_{h,2} \bar{u} = 0 \cdot 0,002 = 0$$



sustav jednačbi i njegovo rješenje:

$$10,9053 \cdot 10^{-4} \cdot X_1 - 2,16049 \cdot 10^{-4} \cdot X_2 + 16 \cdot 10^{-4} = 0$$

$$-2,16049 \cdot 10^{-4} \cdot X_1 + 0,617284 \cdot 10^{-4} \cdot X_2 = 0$$

$$X_1 = -4,78526 \quad \text{el} \quad X_2 = -16,7484$$

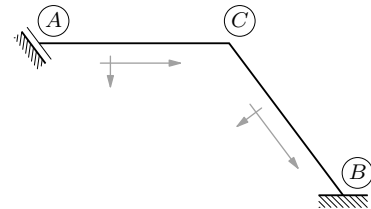
vrijednosti momenata savijanja:

$$M(\xi_i) = X_1 m_1(\xi_i) + X_2 m_2(\xi_i)$$

$$\begin{aligned} M_A &= -4,78526 \cdot 0 + (-16,7484) \cdot (-1) \\ &= 16,7484 \text{ kN} \end{aligned}$$

$$\begin{aligned} M_C &= -4,78526 \cdot 3 + (-16,7484) \cdot (-1) \\ &= 2,39272 \text{ kN} \end{aligned}$$

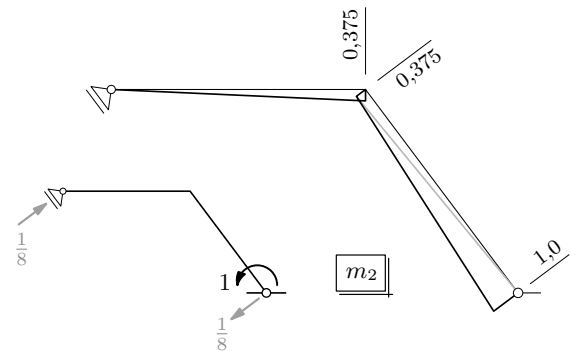
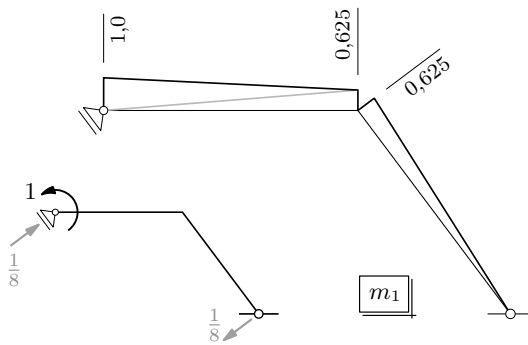
$$\begin{aligned} M_B &= -4,78526 \cdot 8 + (-16,7484) \cdot (-1) \\ &= -21,5336 \text{ kN} \end{aligned}$$



dijagram momenata savijanja prikazan je **na stranici 6**

ili osnovni sistem II.:

dijagrami m_i i koeficijenti popustljivosti $\delta_{i,j}$:



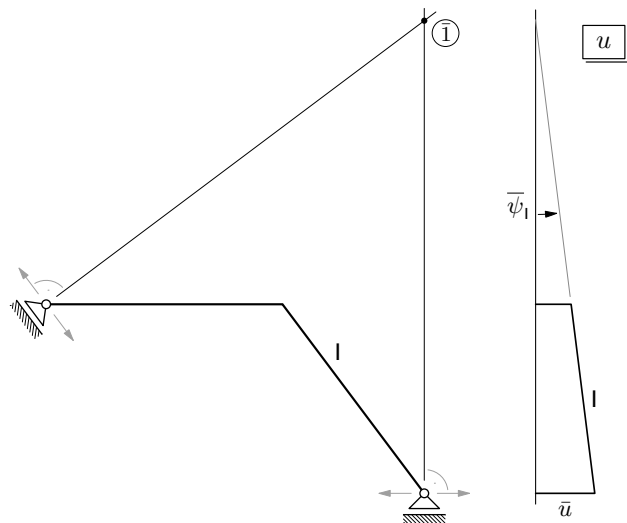
$$\begin{aligned} \delta_{1,1} &= \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 1 \cdot 5 \right) \left(\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 0,625 \right) + \left(\frac{1}{2} \cdot 0,625 \cdot 5 \right) \left(\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 0,625 \right) \right. \\ &\quad \left. + \left(\frac{1}{2} \cdot 0,625 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,625 \right) \right] = 2,47557 \cdot 10^{-5} \end{aligned}$$

$$\begin{aligned} \delta_{2,2} &= \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 0,375 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,375 \right) + \left(\frac{1}{2} \cdot 0,375 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,375 + \frac{1}{3} \cdot 1 \right) \right. \\ &\quad \left. + \left(\frac{1}{2} \cdot 1 \cdot 5 \right) \left(\frac{1}{3} \cdot 0,375 + \frac{2}{3} \cdot 1 \right) \right] = 1,70396 \cdot 10^{-5} \end{aligned}$$

$$\delta_{1,2} = \delta_{2,1} = \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 1 \cdot 5 \right) \left(\frac{1}{3} \cdot 0,375 \right) (-1) + \left(\frac{1}{2} \cdot 0,625 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,375 \right) (-1) + \left(\frac{1}{2} \cdot 0,625 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,375 + \frac{1}{3} \cdot 1 \right) (-1) \right] = -9,966\,56 \cdot 10^{-6}$$

$$\bar{\delta}_1 = 0 \quad \mathcal{E} \quad \bar{\delta}_2 = 0 \quad (\text{nije raskinut spoj na pravcu prisilnoga pomaka})$$

$\delta_{i,0}$ pomoću dijagrama projekcija pomaka na horizontalnu os:



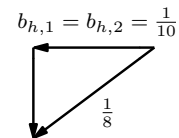
$$\bar{\psi}_1 = \frac{\bar{u}}{10} = 0,000\,2$$

$$\delta_{1,0} = \delta_{2,0} = \bar{\psi}_1 = 0,000\,2$$

$\delta_{i,0}$ pomoću virtualnoga rada:

$$\delta_{1,0} = -b_{h,1} \bar{u} = -\left(-\frac{1}{10} \cdot 0,002 \right) = 0,000\,2$$

$$\delta_{2,0} = -b_{h,2} \bar{u} = 0,000\,2$$



sustav jednačbi i njegovo rješenje:

$$2,475\,57 \cdot 10^{-5} \cdot X_1 - 0,996\,656 \cdot 10^{-5} \cdot X_2 + 20 \cdot 10^{-5} = 0$$

$$-0,996\,656 \cdot 10^{-5} \cdot X_1 + 1,703\,96 \cdot 10^{-5} \cdot X_2 + 20 \cdot 10^{-5} = 0$$

$$X_1 = -16,748\,3 \quad \mathcal{E} \quad X_2 = -21,533\,5$$

vrijednosti momenata savijanja:

$$M_A = (-16,748\,3) \cdot (-1) + (-21,533\,5) \cdot 0 = 16,748\,3 \text{ kN}$$

$$M_C = (-16,748\,3) \cdot (-0,625) + (-21,533\,5) \cdot 0,375 = 2,392\,63 \text{ kN}$$

$$M_B = (-16,748\,3) \cdot 0 + (-21,533\,5) \cdot 1,375 = -21,533\,5 \text{ kN}$$

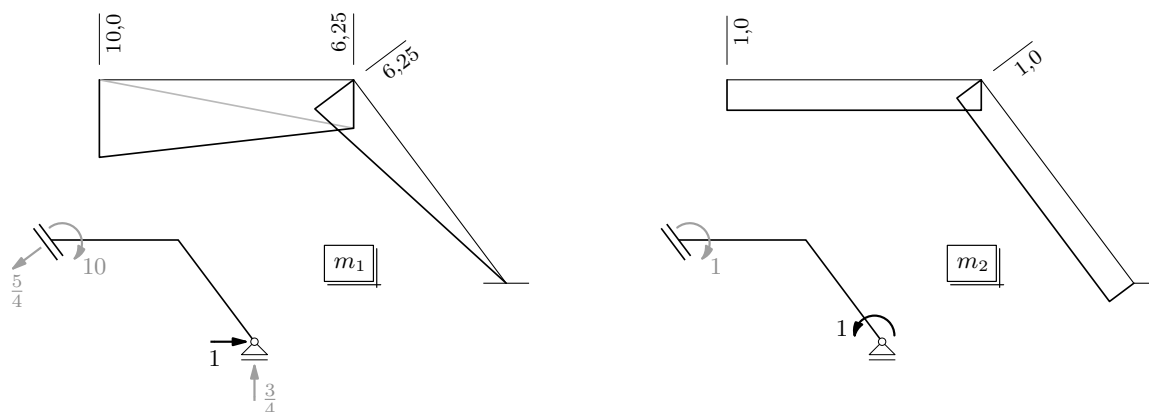
dijagram momenata savijanja prikazan je **na stranici 6**

ili osnovni sistem III.:

$$\bar{\delta}_1 = \bar{u} = 0,002 \quad \ell' \quad \bar{\delta}_2 = 0 \quad (\text{raskinut je spoj na mjestu i na pravcu prisilnoga pomaka})$$

$$\delta_{1,0} = 0 \quad \ell' \quad \delta_{2,0} = 0 \quad (\text{prisilni pomak ne utječe na osnovni sistem})$$

dijagrami m_i i koeficijenti popustljivosti $\delta_{i,j}$:



$$\delta_{1,1} = \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 10 \cdot 5 \right) \left(\frac{2}{3} \cdot 10 + \frac{1}{3} \cdot 6,25 \right) + \left(\frac{1}{2} \cdot 6,25 \cdot 5 \right) \left(\frac{1}{3} \cdot 10 + \frac{2}{3} \cdot 6,25 \right) + \left(\frac{1}{2} \cdot 6,25 \cdot 5 \right) \left(\frac{2}{3} \cdot 6,25 \right) \right] = 2,475\,57 \cdot 10^{-3}$$

$$\delta_{2,2} = \frac{2}{EI} [(1,0 \cdot 5) \cdot 1,0] = 6,172\,84 \cdot 10^{-5}$$

$$\delta_{1,2} = \delta_{2,1} = \frac{1}{EI} \left[(1 \cdot 5) \left(\frac{1}{2} \cdot 10 + \frac{1}{2} \cdot 6,25 \right) + (1 \cdot 5) \left(\frac{1}{2} \cdot 6,25 \right) \right] = 3,472\,22 \cdot 10^{-4}$$

sustav jednačbi i njegovo rješenje:

$$24,755\,7 \cdot 10^{-4} \cdot X_1 + 3,472\,22 \cdot 10^{-4} \cdot X_2 = 20 \cdot 10^{-4}$$

$$3,472\,22 \cdot 10^{-4} \cdot X_1 + 0,617\,284 \cdot 10^{-4} \cdot X_2 = 0$$

$$X_1 = 3,828\,13 \quad \ell' \quad X_2 = -21,533\,2$$

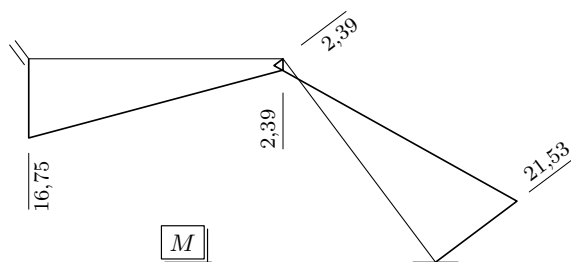
vrijednosti momenata savijanja:

$$M_A = 3,828\,13 \cdot 10 + (-21,533\,2) \cdot 1 = 16,748\,1 \text{ kN}$$

$$M_C = 3,828\,13 \cdot 6,25 + (-21,533\,2) \cdot 1 = 2,392\,61 \text{ kN}$$

$$M_B = 3,828\,13 \cdot 0 + (-21,533\,2) \cdot 1 = -21,533\,2 \text{ kN}$$

dijagram momenata savijanja:



dijagrami poprečnih i uzdužnih sila:

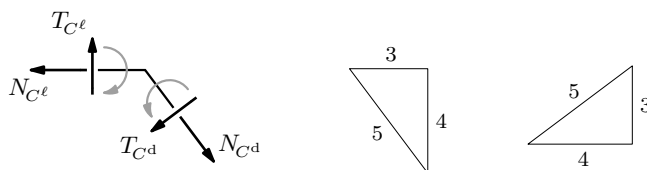
vrijednosti poprečnih sila:

$$T(\xi_i) = \frac{dM(\xi_i)}{d\xi_i}$$

$$T_A = -\frac{16,7481 - 2,39261}{5} = -2,87112 \text{ kN} = T_{C\text{lijevo}}$$

$$T_{C\text{desno}} = -\frac{2,39261 - (-21,5332)}{5} = -4,78516 \text{ kN} = T_B$$

vrijednosti uzdužnih sila:



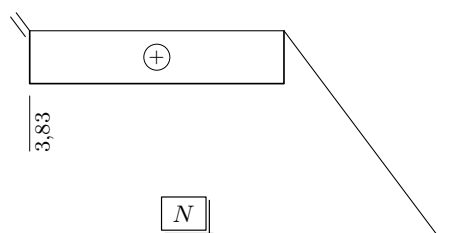
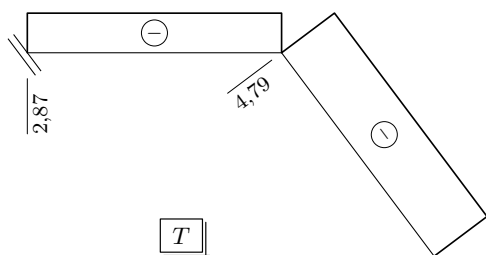
$$-T_{C\text{lijevo}} + \frac{3}{5} T_{C\text{desno}} + \frac{4}{5} N_{C\text{desno}} = 0 \quad (\text{ravnoteža projekcija sila u čvoru 2 na os } z)$$

$$N_{C\text{desno}} = \frac{5}{4} \left(T_{C\text{lijevo}} - \frac{3}{5} T_{C\text{desno}} \right) = 0,000024 \simeq 0 = N_B$$

$$-N_{C\text{lijevo}} + \frac{4}{5} T_{C\text{desno}} - \frac{3}{5} N_{C\text{desno}} = 0 \quad (\text{ravnoteža projekcija sila u čvoru 2 na os } x)$$

$$N_{C\text{lijevo}} = \frac{4}{5} T_{C\text{desno}} = 3,828128 \text{ kN} = N_A$$

dijagrami:



deformacijska kontrola:

(ako ne znate što je to: *Rješenje zadatka C1. druga kolokvija 2022./2023. godine*

(<http://master.grad.hr/nastava/g1/g11/isp/1-23-2k-c1.pdf>))

uzmimo da je dijagram M nacrtan pomoću osnovnoga sistema I.:

provjera pomoću osnovnoga sistema II.:

$$\begin{aligned}\varphi_B &= \varphi_B(M) + \varphi_B(\bar{u}) && (\varphi_B(\bar{u}) — \text{utjecaj prisilnoga pomaka}) \\&= \sum \int \frac{M(s) m_2(s)}{EI(s)} ds + \bar{\psi}_1 && (m_2 — \text{desni dijagram na slici na stranici 3} \\&&& \bar{\psi}_1 — \text{dijagram projekcija pomaka na stranici 4}) \\&= \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 16,75 \cdot 5 \right) \left(\frac{1}{3} \cdot 0,375 \right) + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,375 \right) \right. \\&\quad + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \left(\frac{1}{3} \cdot 1,0 + \frac{2}{3} \cdot 0,375 \right) \\&\quad \left. + \left(\frac{1}{2} \cdot 21,53 \cdot 5 \right) \left(\frac{2}{3} \cdot 1,0 + \frac{1}{3} \cdot 0,375 \right) (-1) \right] + 0,0002 \\&= -0,000199987 + 0,0002 = 0,000000013 \simeq 0 \\&\left[\text{relativna pogreška: } \varepsilon_r = \frac{0,000000013}{0,0002} = 0,0065\% \right]\end{aligned}$$

ili:

$$\begin{aligned}\varphi_A &= \varphi_A(M) + \varphi_A(\bar{u}) && (\varphi_A(\bar{u}) — \text{utjecaj prisilnoga pomaka}) \\&= \sum \int \frac{M(s) m_1(s)}{EI(s)} ds + \bar{\psi}_1 && (m_1 — \text{lijevi dijagram na slici na stranici 3}) \\&= \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 16,75 \cdot 5 \right) \left(\frac{2}{3} \cdot 1,0 + \frac{1}{3} \cdot 0,625 \right) (-1) \right. \\&\quad + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \left(\frac{1}{3} \cdot 1,0 + \frac{2}{3} \cdot 0,625 \right) (-1) \\&\quad \left. + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \left(\frac{2}{3} \cdot 0,625 \right) (-1) + \left(\frac{1}{2} \cdot 21,53 \cdot 5 \right) \left(\frac{1}{3} \cdot 0,625 \right) \right] \\&\quad + 0,0002 \\&= -0,000199987 + 0,0002 = 0,000000013 \simeq 0\end{aligned}$$

ili provjera pomoću osnovnoga sistema III.:

$$u_B = \sum \int \frac{M(s) m_1(s)}{EI(s)} ds \quad (m_1 — \text{lijevi dijagram na slici na stranici 5})$$

$$\begin{aligned}
&= \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 16,75 \cdot 5 \right) \left(\frac{2}{3} \cdot 10,0 + \frac{1}{3} \cdot 6,25 \right) \right. \\
&\quad + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \left(\frac{1}{3} \cdot 10,0 + \frac{2}{3} \cdot 6,25 \right) \\
&\quad \left. + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \left(\frac{2}{3} \cdot 6,25 \right) + \left(\frac{1}{2} \cdot 21,53 \cdot 5 \right) \left(\frac{1}{3} \cdot 6,25 \right) (-1) \right] \\
&= 0,001\,999\,87 \text{ m} = 1,999\,87 \text{ mm} \simeq 2 \text{ mm} = \bar{u}
\end{aligned}$$

$$\left[\text{relativna pogreška: } \varepsilon_r = \frac{|1,999\,87 - 2|}{2} = 0,006\,5\% \right]$$

ili:

$$\begin{aligned}
\varphi_B &= \sum \int \frac{M(s) m_2(s)}{EI(s)} ds \quad (m_1 \text{ — desni dijagram na slici na stranici 5}) \\
&= \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 16,75 \cdot 5 \right) \cdot 1 + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \cdot 1 + \left(\frac{1}{2} \cdot 2,39 \cdot 5 \right) \cdot 1 \right. \\
&\quad \left. + \left(\frac{1}{2} \cdot 21,53 \cdot 5 \right) \cdot 1 \cdot (-1) \right] = 0
\end{aligned}$$

(budući da je osnovni sistem III. oblikovan raskidanjem spoja na mjestu i na pravcu prisilnoga pomaka, taj pomak nema utjecaja)